ETME 3120
Maintenance of Mechatronic Systems

Lesson 2: Statistical Applications
Refer to Chapter 2 in the textbook

Reference: Productivity and Reliability-Based Maintenance Management, M. Stephens, 2010

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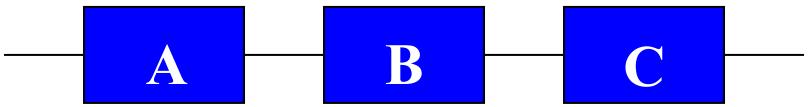
Reliability

- **Reliability:** The ability of a system to perform its intended function during its expected life period.
- The probability that the system will perform its specified function under the specified conditions throughout its specified life expectancy.
- It is important to understand the statistical reliability of the system design in order to plan for an adequate program.
- Most operation equipment or facilities are treated as a system rather than an individual component.

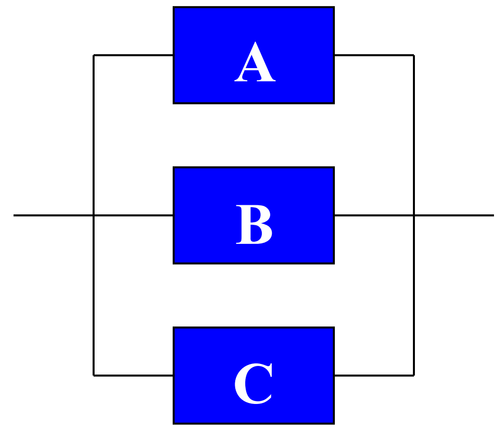
System Reliability

System Configurations:

Series components:



Parallel components:



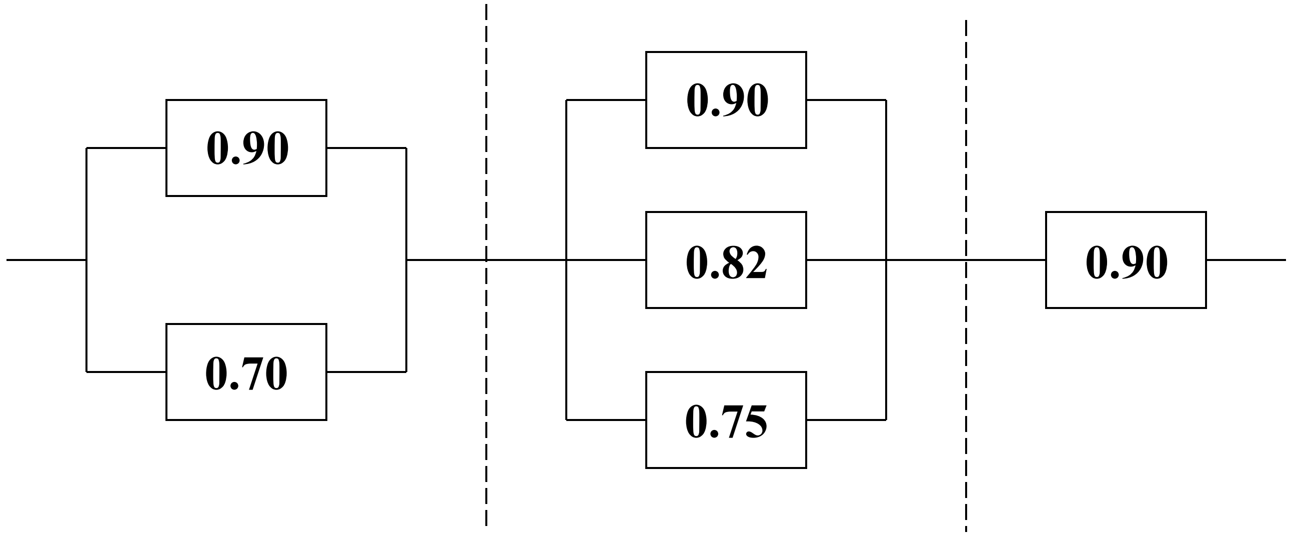
For Series Components: $R_S = R_1 \times R_2 \times R_3 \ldots \times R_n$

Thus, $R_s = R_A \times R_B \times R_C$

For Parallel Components: $R_P = 1 – [(1 - R_1)(1 - R_2) \ldots (1 – R_n)]$

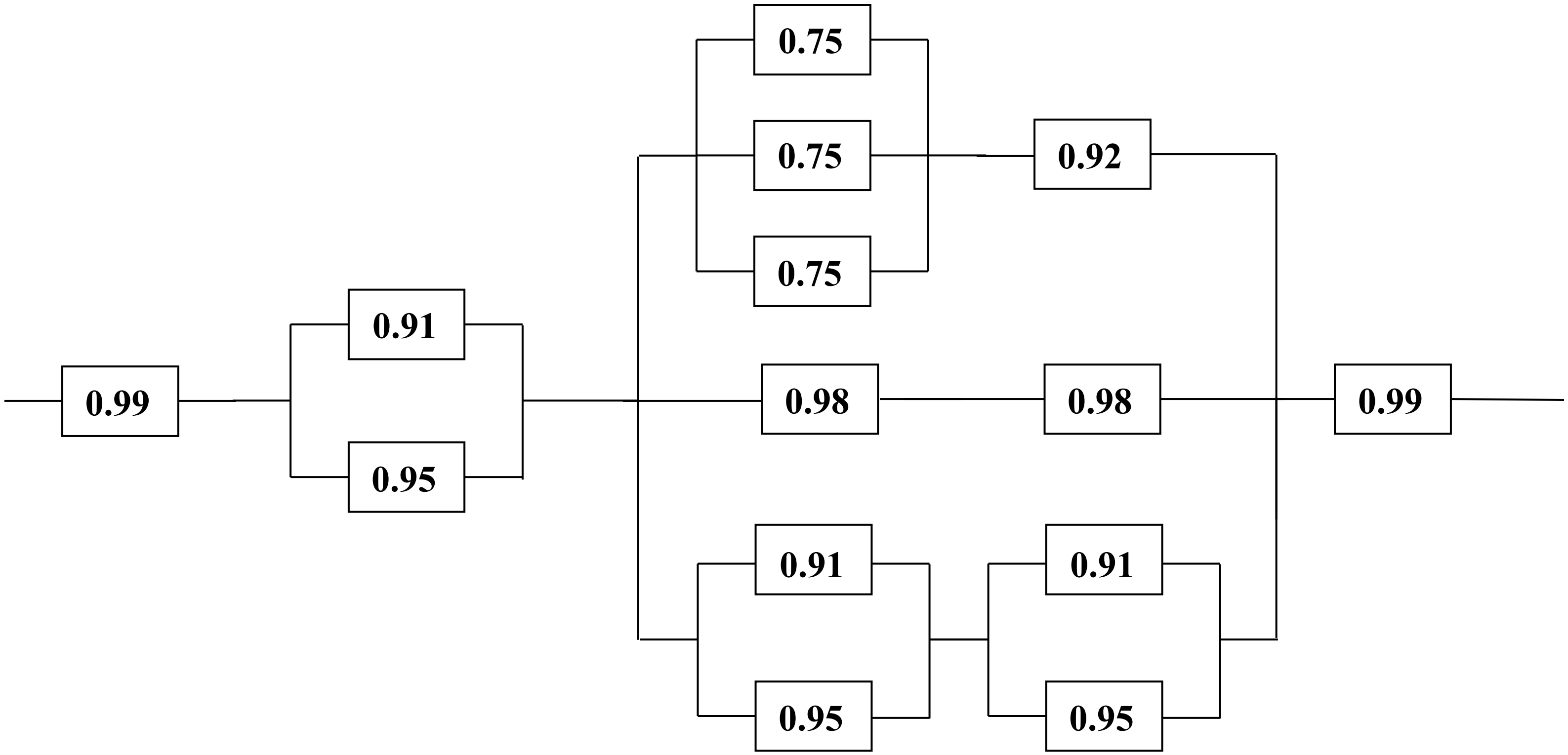
Thus, $R_p = 1-(1-R_A)(1- R_B)(1- R_C)$

Example 1:



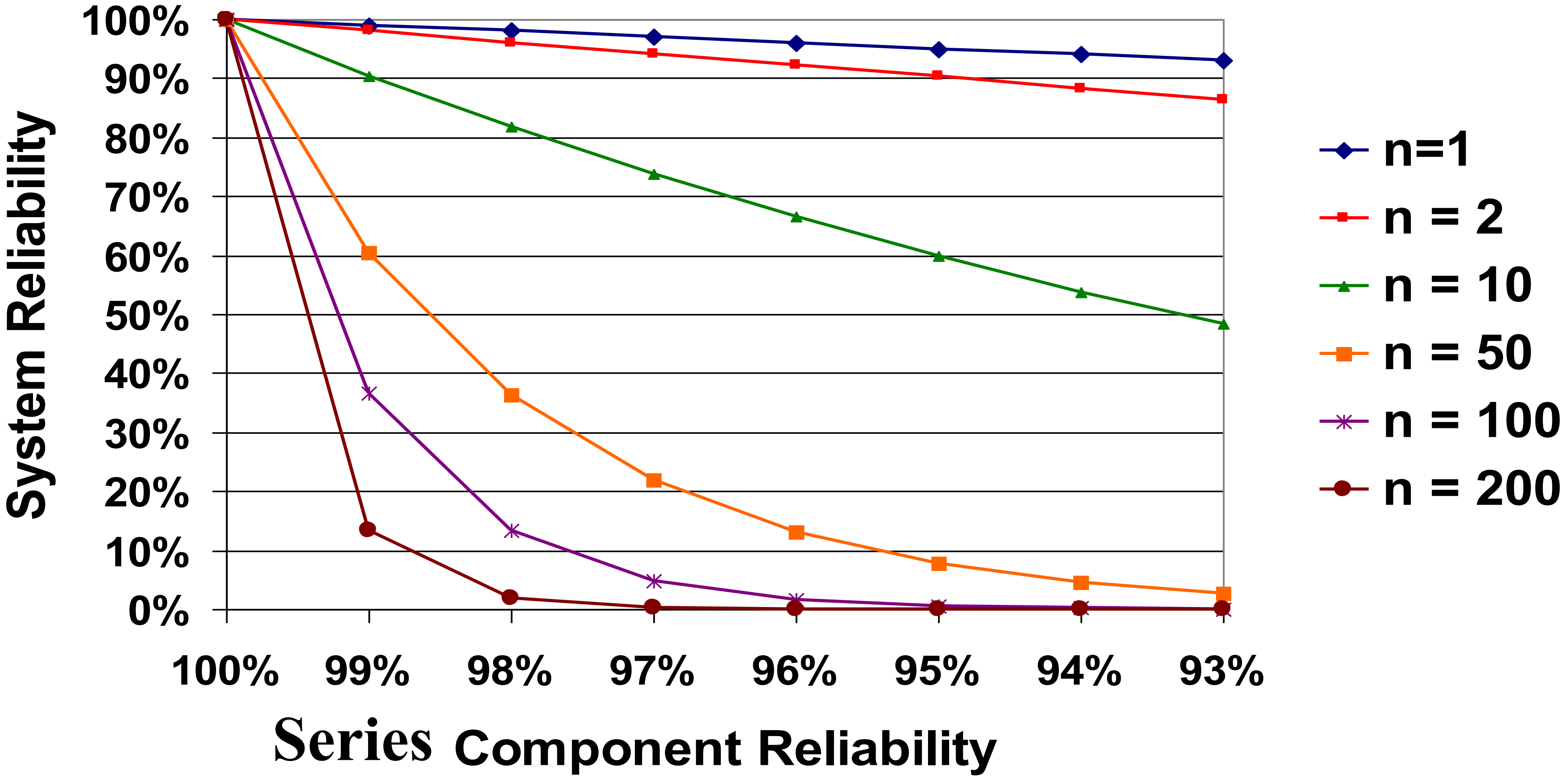
- Find the system reliability:
- Solution:
- Rp1 = 1-(1-0.90)(1-0.70) = 0.97
- Rp2 = 1-(1-0.90)(1-0.82)(1-0.75) = 0.9955
- Rs = 0.97 x 0.9955 x 0.90 = 0.87

Exercise:



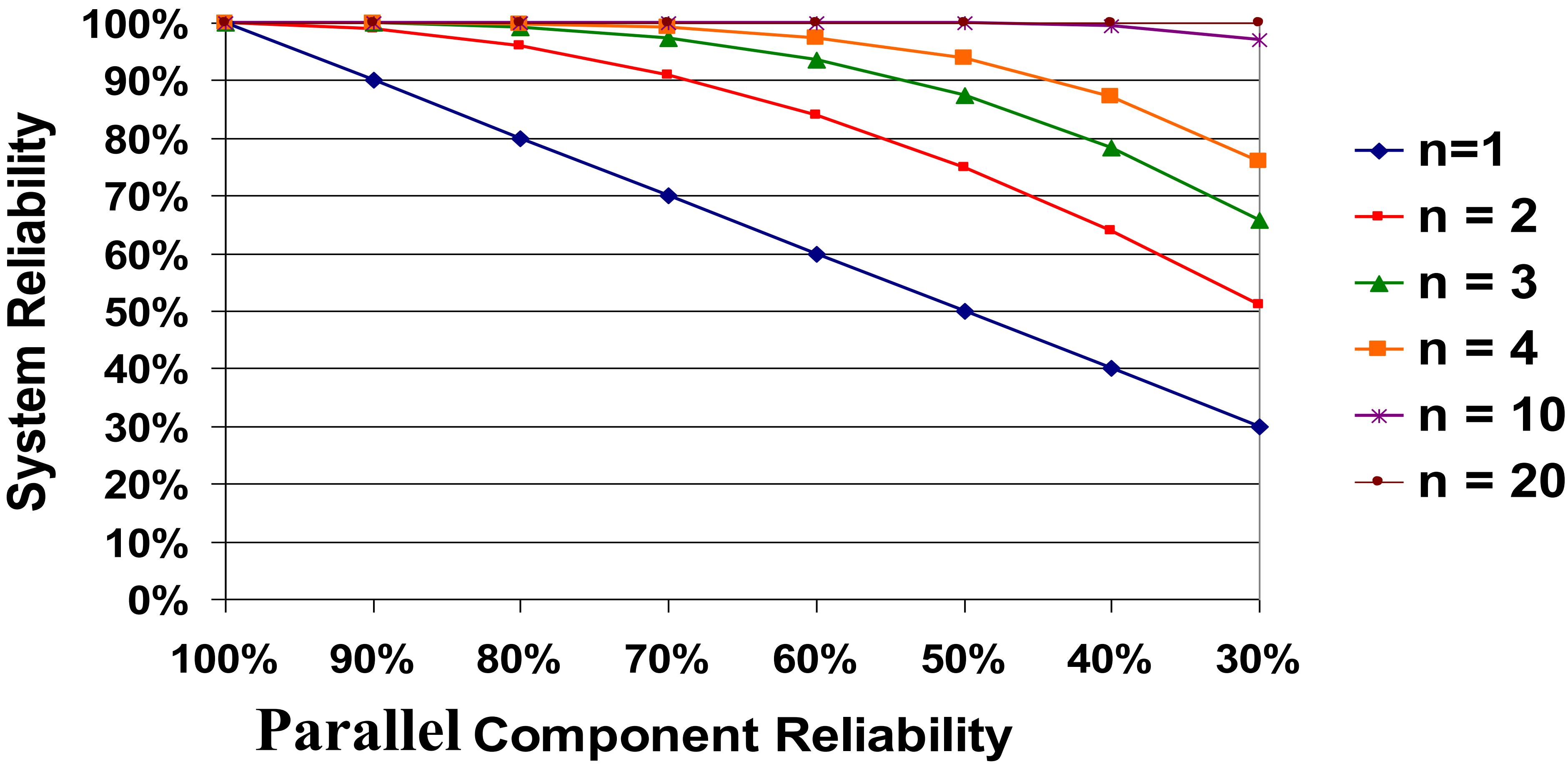
- Find the system reliability:

Reliability as a function of number of components



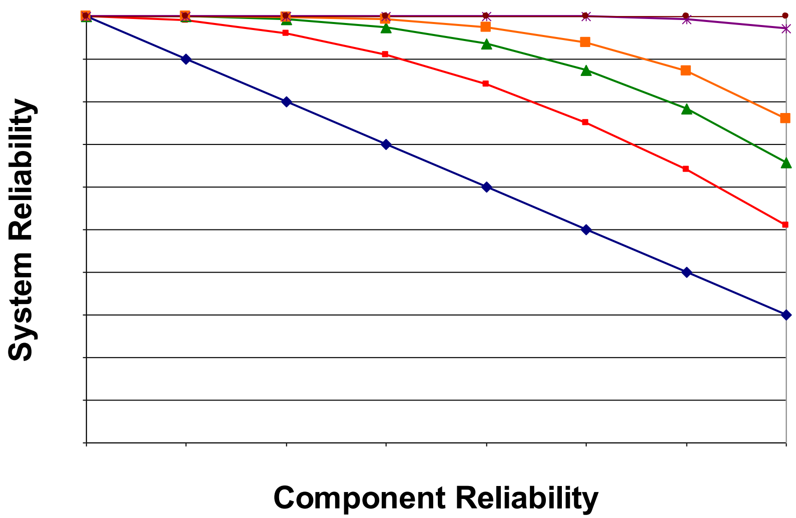
Here system reliability falls significantly as the number of series components increases. For example, at n= 200 components, and each component has a reliability of 98%, the overall reliability is less than 2%

Parallel Systems

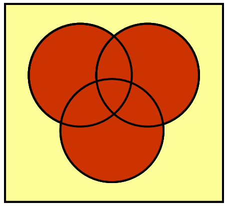


Here system reliability falls significantly as the number of series components increases. For example, at n= 4 components, and each component has a reliability of 80%, the overall reliability is almost 100% (exactly 99.84%)

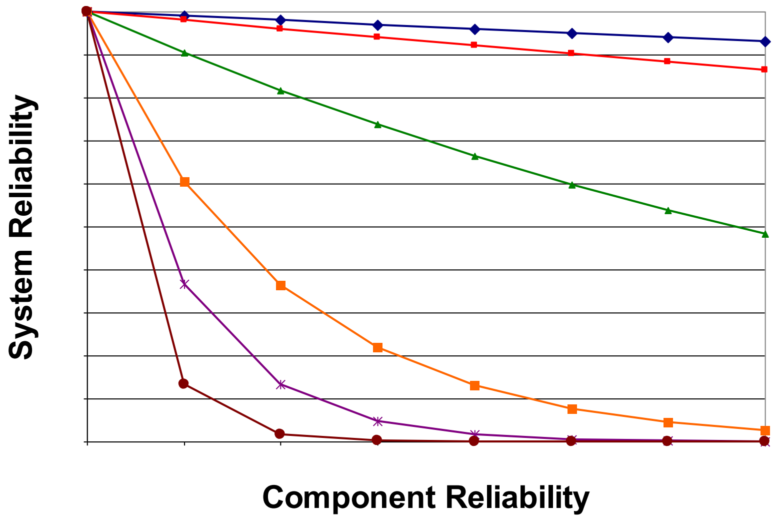
Parallel



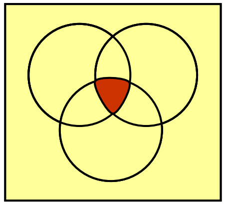
For the parallel configuration, the overall reliability of the system is greater than the reliability of each individual component.



Series



For the series configuration, the overall reliability of the system is smaller than the reliability of each individual component.



Failure and Failure Rate

- Failure: The absence of reliability
- Failure rate (**λ**): The ratio of the number of failures to the total number of item-hours tested. Assuming that Reliability does not change with time, then:

$$\boldsymbol{R = 1 - \lambda}$$

$$\boldsymbol{\lambda} = \frac{\text{Number of failures}}{\text{Number of item-hours tested}}$$

Example 2:

Ten instruments are tested for a period of 100 hours. If four units failed after 23, 42, 59, and 82 hours respectively. What is the failure rate of this instrument? What is the instrument's reliability (assume it doesn't change with time)?

- $\boldsymbol{\lambda} = \frac{4}{23+42+59+82+(6\times100)} = 0.0050 = 0.50\%$
- $\mathrm{R} = 1 - \lambda = 1 - 0.0050 = 0.995 = 99.50\%$

MTBF and MTTF

- **Mean Time Between Failures (MTBF)** is the expected average time, or the expected frequency by which we can expect the equipment to fail.
- MTBF is estimated based on historical data and statistical analysis
- **Mean Time to Failure (MTTF):**
- **What is Difference between MTBF and MTTF?**
- MTBF refers to the situation when a failure is temporary and the unit can be repaired and brought back online by fixing or replacing specific components.
- MTTF refers to units or components that are disposed off upon failure, Therefore, MTTF can be simply equated to the life expectancy of a unit.

Mean Down Time (MDT)

- The mean downtime (MDT) or mean time to repair (MTTR) is the average total amount of time that it takes to return a failed equipment back on-line and ready for operation
- The MDT includes issuing a work order, dispatching maintenance crew, and completion of the tasks.

$$MTBF(in\ hours) = \frac{1}{\lambda}$$

Remember that: $R = 1 - \lambda$

(Assuming that reliability stays constant with time)

$$Availability = \frac{MTBF}{MTBF + MDT}$$

Availability = Proportion of time that the equipment is expected to be operational

λ = Failure rate

MTBF = Mean time between failure

MDT = Mean downtime

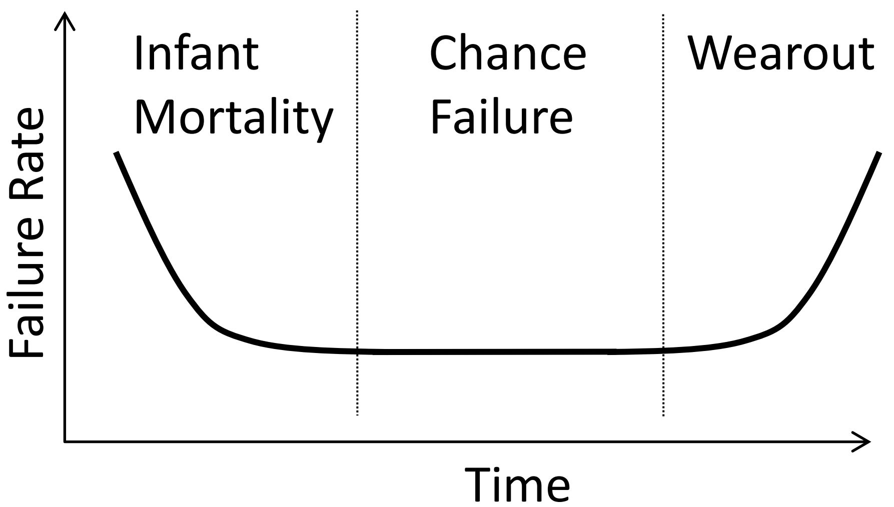
R = Reliability

Example 3

- Five robot units were produced by a team of students and were tested for a period of 40 hours. If four of the units failed after 10, 22, 24, and 31 hours, respectively, calculate the robot's failure rate, the reliability (assume reliability stays constant with time), the MTBF, and availability if it takes five work hours to fix a robot unit.
- Solution:
- Failure rate $\lambda = \frac{4}{10+22+24+31+40} = 0.0315$
- Thus Reliability $R = 1 - \lambda = 0.9685 = 96.85\ \%$
- $MTBF = \frac{1}{\lambda} = \frac{1}{0.0315} = 31.75\ hours$
- Availability $= \frac{\text{MTBF}}{\text{MTBF+MDT}} = \frac{31.75}{31.75+5} = 0.864 = 86.4\ \%$

Life-Cycle Failure Rate

- Failure occurs at different rates during the life of the product and follows different statistical and probability distributions.
- Typically, the failure rate as a function of the equipment's lifetime, follows the "bathtub" curve below in three stages:
- Stage 1: Infant Mortality (Debugging stage): Here failures are high and mostly due to defects, design or installation errors.
- Stage 2: Chance Failure, here failures are random.
- Stage 3: Wear-out phase



Reliability as a function of time (Slide 1)

- The failure rate is constant, so it does not change with time.
- Reliability decreases exponentially with time.
- The risk of failure is the probability of a failure occurrence. It complements reliability. Thus:

$$\boldsymbol{R(t) = e^{-\lambda t} = 1 - Failure\ Probability}$$

- **Example 4:** What is the reliability of the robotic units in example 3 at the following time intervals:

  a) At t = 1 hour

  b) At t= MTBF

  Solution: a) Reliability $R(1) = e^{-0.0315 \cdot 1} = 0.9689$

  Note that the reliability at t= 1, R(1) is equivalent to $1 - \lambda$

  b) MTBF $= 31.75$ hours, thus:

  $R(MTBF) = R(31.75) = e^{-0.0315 \cdot 31.75} = 0.3678 = 36.78\ \%$

  At MTBF, the reliability of the equipment becomes 36.78% and the probability of failure of the equipment becomes 63.22 %

Reliability as a function of time (Slide 2)

- **Example 5:** Seven units of equipment have been tested for 100 hours. Four units failed after 30, 34, 55, and 67 hours. If the average downtime is 40 hours, Calculate:

  a) Failure rate   b) MTBF

  c) Availability of the equipment

  d) Reliability at the first hour of operation

  e) Reliability at time = half of MTBF

  Solution:

  a) $\lambda = 4/(30 + 34 + 55 + 67 + 300)$ =0.0082

  b) MTBF = 1/ λ = 1/0.0082 = 121.5 hours

  c) Availability = 121.5 / (121.5+40) = 0.752

  d) $R(1) = e^{-0.0082\cdot 1} = 0.9918$

  e) $R\left(\frac{MTBF}{2}\right) = R(60.75) = e^{-0.0082\cdot 60.75} = 0.6077$

We can see from the example that reliability of the equipment dropped from 99.18 % at the start to 60.77 % after 60.75 hours of operation.

Question: What is the reliability at t = 121.5 hours?

Reliability as a function of time (Slide 3)

Example 6: In example 5, what should be the Preventive maintenance interval if the failure probability should be no more than 25%

Solution:

R(t) = 1- Failure Probability = 1-0.25 = 0.75

But $R(t) = e^{-\lambda t}$

- Therefore: $0.75 = e^{-\lambda t}$ $thus: 0.75 = e^{-(0.0082 \cdot t)}$

Taking the natural logarithm of both sides:

$$ln 0.75 = ln\ (e^{-(0.0082 \cdot t)})$$

Thus: $-0.2876 = -0.0082t\ ln(e)$

Which results that: t = 35 hours.

From the above example, we can see that if preventive maintenance takes place every 35 hours of operation, then the risk of failure of the equipment will be 25% .

If the preventive maintenance interval is the MTBF which is 121.5 hours, then the risk of failure is *1-R(MTBF)* which is 63.21 %

Maintenance and Risk Economics (Slide 1)

- To determine whether a preventive maintenance(PM) program is economical or not, the cost for having one or not needs to be calculated.
- The economy is determined based on the following information:
  - The cost per failure occurrence of the equipment
  - The cost per PM routine
  - The number of work hours per year of the equipment
- Based on the three parameters above, a PM program can be adjusted for maximum economic benefit.

Maintenance and Risk Economics (Slide 2)

Example 7: Assume that a machine that has a MTBF of 240 hours, is operated 350 days per year for 12 hours per day. The cost of failure occurrence is $3,000. A PM program would cost $300 per routine. What is the cost saving of having a PM routine every 200 hours of operation?

Solution:

Total hours of operation = $350 \times 12 = 4{,}200$ hours

At MTBF of 240 hours, $\lambda = \frac{1}{MTBF} = 0.004167$

At time intervals of t = 200 hours, $R(200) = e^{-0.004167 \cdot 200} = 0.4346$ = 43.46% thus there is a 56.54 % risk of failure between scheduled PM routines.

Number of PM routines per year = 4,200 / 200 = 21 routines.

Therefore, Number of failures = $21 \times 0.5654 = 11.87 = 12\ failures.$

Hence, total cost $= 21 \times \$300 + 12 \times \$3{,}000 = \mathbf{\$42,300}$

If no PM routine at all, then the total number of failures will be:

$$Total\ failures\ without\ PM = \frac{4{,}200}{MTBF} = \frac{4{,}200}{240} = 17.5\ failures$$

Total cost if no PM program = $17.5 \times \$3{,}000 = \$52{,}500$

Therefore, the PM program at 200 hour intervals saves at least $10,200